AP Calculus AB

Derivatives of Exp & Log

1) $y = 2e^x$

$\frac{dy}{dx} = 2e^x$

2) $y = e^{2x}$

$y' = e^{2x} \cdot d[2x]$

$y' = 2e^{2x}$

3) $y = \log_3(\sin x)$

$y' = \frac{d[\sin x]}{[\sin x] \cdot \ln 3}$

$y' = \frac{\cos x}{(\sin x) \cdot \ln 3} = \frac{\cot x}{\ln 3}$

4) $y = e^{-5x}$

$y' = e^{-5x} \cdot d[-5x]$

$y' = -5e^{-5x}$

5) $y = e^{2x/3}$

$y' = e^{2x/3} \cdot d[2x/3]$

$y' = \frac{2}{3} e^{2x/3}$

6) $y = \ln(x^3)$

$y' = \frac{d[x^3]}{[x^3]}$

$y' = \frac{3x^2}{x^3}$

$y' = \frac{3}{x}$

7) $y = [x][e^x]$

$y' = xe^x + e^x$

$y' = e^x(x+1)$

8) $y = x^2e^x - xe^x$

$y = [e^x](x^2 - x)$

$y' = [e^x]\, d[x^2-x] + [x^2-x]\, d[e^x]$

$y' = e^x(2x-1) + (x^2-x) \cdot e^x$

$y' = e^x[2x - 1 + x^2 - x]$

$y' = e^x(x^2+x-1)$

9) $y = 4^{\sqrt{x}} = 4^{[x^{1/2}]}$

$y' = 4^{[\ ]} \cdot d[\ ] \cdot \ln 4$

$y' = 4^{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2} \cdot \ln 4$

$y' = \frac{4^{\sqrt{x}} \cdot \ln 4}{2\sqrt{x}}$

10) $y = 5^{[x^2]}$

$y' = 5^{[x^2]} \cdot d[x^2] \cdot \ln 5$

$y' = 5^{x^2} \cdot 2x \cdot \ln 5$

11) $y = \ln(\cos x)$

$y' = \dfrac{d[\cos x]}{[\cos x]}$

$y' = \dfrac{-\sin x}{\cos x}$

$y' = -\tan x$

12) $y = (\ln x)^2$

$y' = 2[\ln x] \cdot d[\ln x]$

$y' = 2\ln x \cdot \frac{1}{x}$

$y' = \dfrac{2\ln x}{x}$

13) $y = (\ln x + \sin x)^4$

$y' = 4[\ln x + \sin x]^3 \cdot d[\ln x + \sin x]$

$y' = 4(\ln x + \sin x)^3 \cdot \left(\frac{1}{x} + \cos x\right)$

14) $y = \ln\left(\frac{10}{x}\right) = \ln[10x^{-1}]$

$y' = \dfrac{d[10x^{-1}]}{[10x^{-1}]}$

$y' = \dfrac{-10x^{-2}}{10x^{-1}}$

$y' = -\dfrac{1}{x}$

15) $y = \log_4[x^2 - 3x + \cos x]$

$y' = \dfrac{d[x^2-3x+\cos x]}{[x^2-3x+\cos x] \cdot \ln 4}$

$y' = \dfrac{2x - 3 - \sin x}{(x^2-3x+\cos x)\ln 4}$

16) $y = [x]\ln x - x$

$y' = x \cdot \frac{1}{x} + \ln x - 1$

$y' = 1 + \ln x - 1 = \ln x$

17) $y = x^2 e^x$

$y' = x^2 e^x + 2x e^x = e^x(x^2+2x)$

$y'' = e^x(2x+2) + e^x(x^2+2x)$

$y'' = e^x(x^2+4x+2)$

18) a) $f(x) = 3x^2 - \ln x$; $x = 1$

point: $f(1) = 3$

slope: $f'(x) = 6x - \frac{1}{x}$

$f'(1) = 5$

$y - 3 = 5(x-1)$

b) $f(x) = \ln(1+\sin x)$; $x = \frac{\pi}{4}$

point: $\left(\frac{\pi}{4}, \ln\left(1+\frac{\sqrt{2}}{2}\right)\right)$

slope: $y' = \dfrac{\cos x}{1+\sin x}$

$y' = \dfrac{\sqrt{2}/2}{1+\sqrt{2}/2} \rightarrow M$

$y - \ln\left(1+\frac{\sqrt{2}}{2}\right) = M\left(x - \frac{\pi}{4}\right)$